Stochastics and Real World Models 2013

Annual joint workshop of the International Graduate College
"Stochastics and Real World Models"
Beijing – Bielefeld

ZiF Bielefeld
July 15–19, 2013

Spokesmen of the IGK:
Zhi-Ming Ma (Beijing), Michael Röckner (Bielefeld)

Organizers:
Frank Riedel (Bielefeld), Rebecca Reischuk (Bielefeld), Pedro Vidal (Bielefeld)

General Information

Center for Interdisciplinary Research, ZiF

The ZiF campus is located behind the university building. Main lectures will take place in the ZiF Plenarsaal. Monday Parallel sessions will be organized at the University in the seminar rooms **T2-137**, **T2-208** and **T2-238**. Wednesday Parallel sessions will be organized at the University in the seminar rooms **U2-216**, **U2-223** and **U2-228**.

The university building

The university is located in one large building. There is one main hall in the middle connecting all parts of the building. These are labelled by letters. The main hall is on level 0 and it houses shops (books, stationaries, grocery), a post office, and several restaurants and coffee shops. All rooms in the University are labelled like **T2-137**. This means the room is in **part T, 2nd floor and has number 137**.

Restaurants in the university building

Westend, opening hours: Mo – Fr 11:00 – 16:00, Sat 11 – 14:30
Located next to the swimming pool at one end of the main hall.
Serves full meals but also cakes and salads. Self-service.

Mensa, opening hours: Mo – Fr 11:30 – 14:00
The Salad bar is a part of the Mensa offering various types of salads (surprise) and some warm food. Here you can pay with cash. The Mensa offers also 2-4 fixed menus. If you want one of the fixed menus, you cannot pay with cash. You need to decide about the menu and buy a "ticket" beforehand at the counter next to the display.

Univarza–Restaurant, opening hours: Mo – Fr 10:00 – 24:00
Serves full meals, but also pizzas, salads, or just coffee or tea.
Besides the restaurant, there is also a snack bar.

Cafeteria, opening hours: Mo – Fr 8:00 – 20:00, Sat 9:00-14:30
Between the main entrance and the Mensa. Serves full meals, sandwiches and light meals as well as cakes, salads, or just coffee or tea.

Furthermore, there are small coffee shops on the ground and first floor of the main hall.

Internet

During the conference you can use your personal guest-account. Just connect to to **WLAN-Network "guest"**. When you open your web browser you will be asked the username and password which you can find in your registration folder. After authentication, you can use the WLAN connection with any software as usual (e.g. Email, Skype, SSH). You might need to re-authenticate e.g. after waking up your computer from sleep/suspend mode.

Library

On the first floor, around the whole building. Math books and journals are in **part V1**. Entrances to the library are from the first floor of the main hall. To go to the mathematics part use the entrances in part L1 or M1.
Opening hours:

Entrance L1: Mo – Fr 8:00 – 1:00 am, Sat – Sun 9:00 – 22:00.

Entrance M1: Mo – Fr 9:00 – 16:00.

Public Transport

The tram **Stadtbahn Linie 4** connects the University to the city center. **Tickets** for one or four trips can be obtained from the machines at each station. You need tickets **"Preisstufe 1"**, single trip 2.20 Euro, 4 trips 7.60 Euro. Make sure to validate your ticket at the stamp machines inside the tram when boarding.

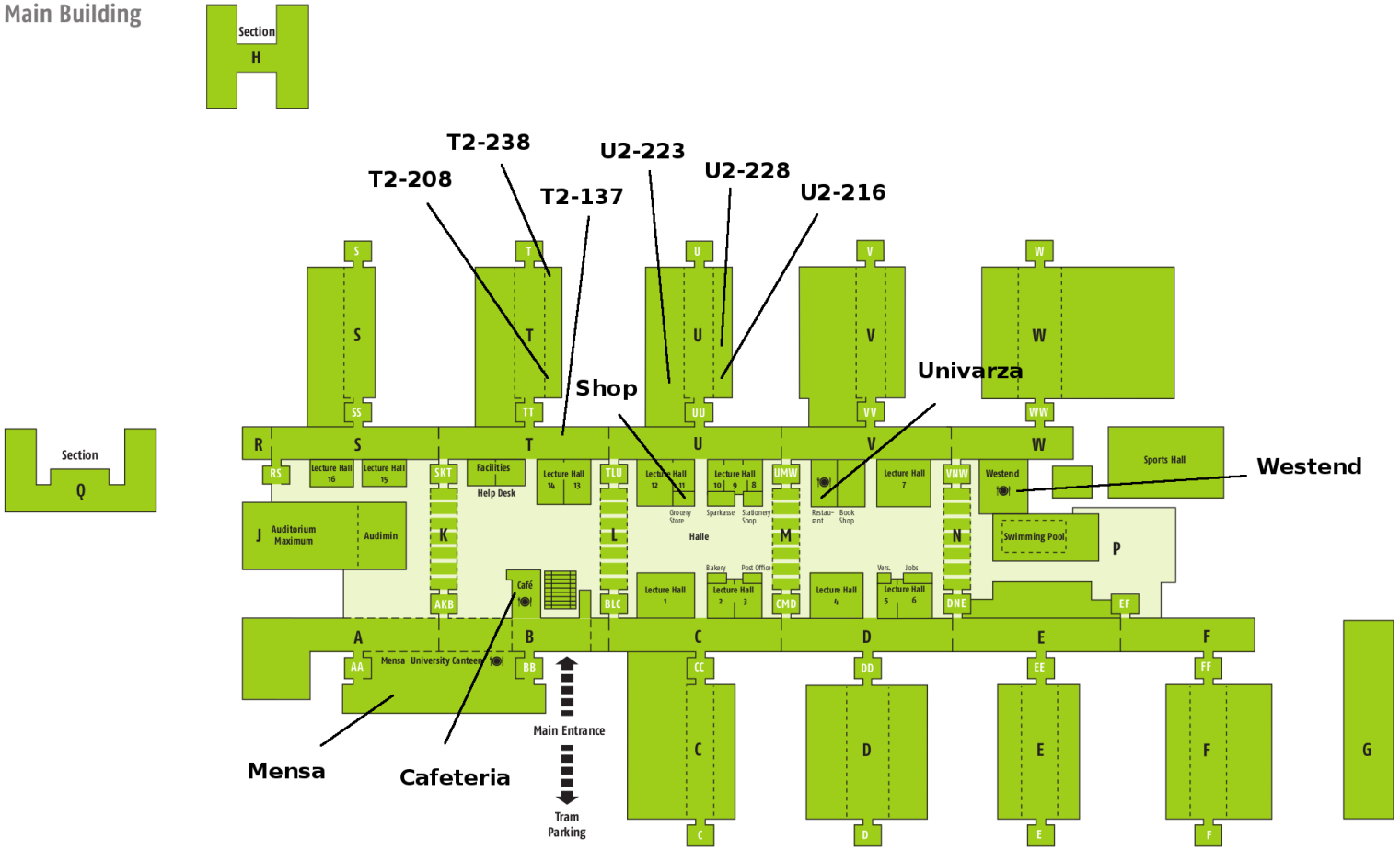
Excursion

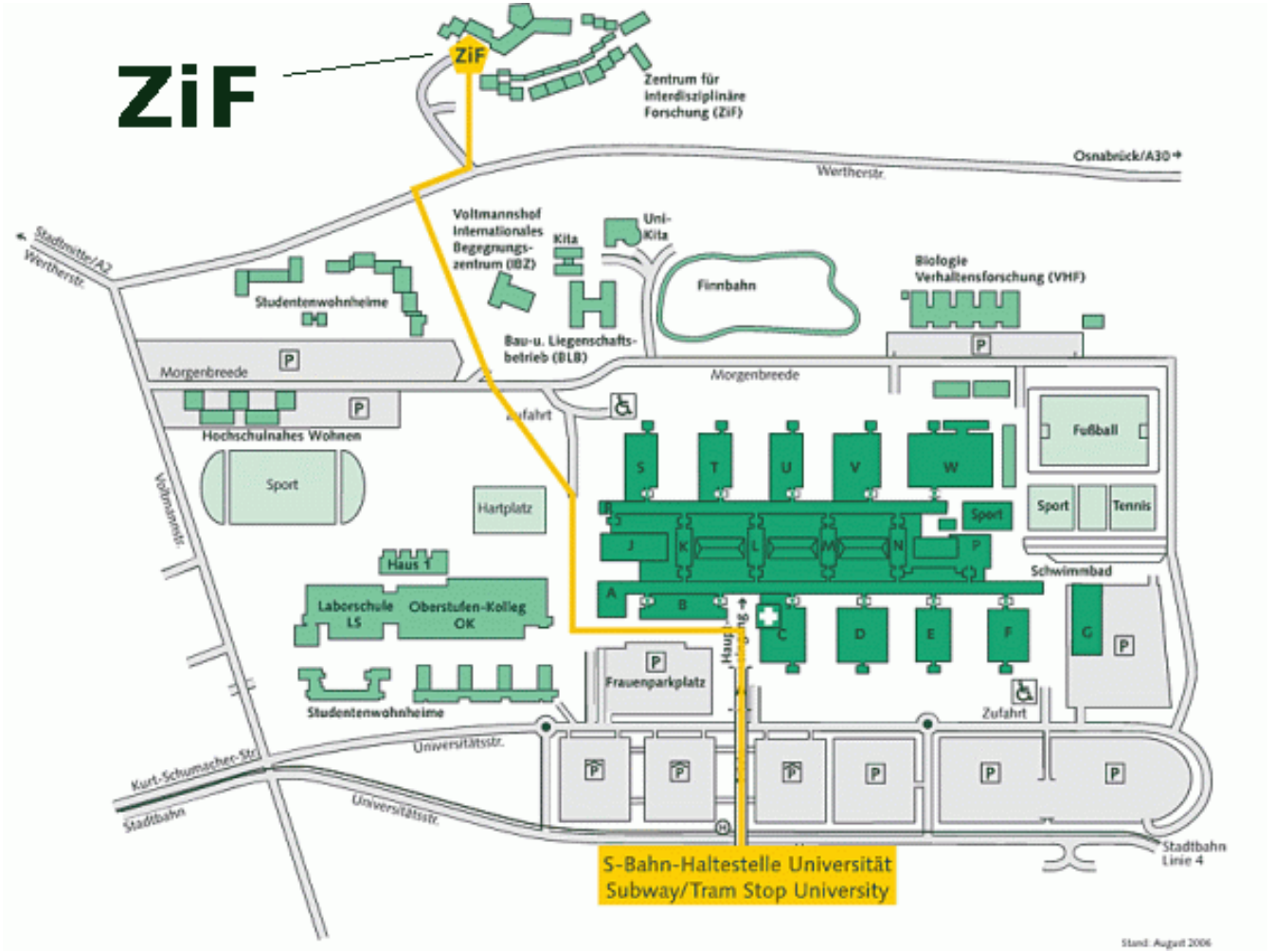
For those who have signed up for the excursion, we will leave Bielefeld University on Tuesday, July 16, after lunch at 2:30pm. The meeting point is outside between building V and W of the University. After a short trip by bus we will arrive at the small and beautiful town of Borgholzhausen. Borgholzhausen lies at the heart of the Teutoburg forest and is famous for its Lebkuchen (German gingerbread). After a short tour in the historical town center, we will briefly discover the Teutoburg forest as we walk towards the Ravensberg castle. Comfortable shoes are recommended. There we will have a special talk as well as a guided tour through this almost thousand year old castle. Finally we will have some refreshments with typical German bread and cake. We expect to be back in Bielefeld around 8pm.

Dinner

The conference dinner will take place on Wednesday, July 17, at Bielefeld's famous Sparrenburg castle. From the hill top Sparrenberg, where the castle is located, you will be able to enjoy not only your dinner, but also the beautiful view over the city of Bielefeld. We will leave the ZiF after the last talk at around 6:30pm and buses will go back to the city center as well as to the University's accommodation afterwards.

Main Building




ZiF
ZiF
Zentrum für Interdisziplinäre Forschung (ZiF)
Osnabrück/A30
Wertherstr.
Stadtmitte/A2
Wertherstr.
Voltmannshof Internationales Begegnungszentrum (IBZ)
Kita
Uni-Kita
Studentenwohnheime
Bau-u. Liegenschaftsbetrieb (BLB)
Finnbahn
Biologie Verhaltensforschung (VHF)
Morgenbreede
Morgenbreede
Hochschulnahes Wohnen
Sport
Hartplatz
Voltmannstr.
Haus 1
Laborschule LS
Oberstufen-Kolleg OK
Studentenwohnheime
Frauenparkplatz
Zufahrt
Fußball
Sport
Tennis
Schwimmbad
Universitätsstr.
Kurt-Schumacher-Str.
Stadtbahn
Universitätsstr.
S-Bahn-Haltestelle Universität
Subway/Tram Stop University
Stadtbahn Linie 4
Stand: August 2006

Monday

8:30–8:50 *– Registration –*

8:50–9:00 *– Opening –*

9:00–9:45 ⇝ Volker Betz
Planar spatial random permutations

9:45–10:30 ⇝ Zhi-Ming Ma
Markov jump processes in modelling coalescent with recombination

10:30–11:15 ⇝ Shunglong Luo
Measuring Quantum Non-Markovianity

11:15–11:45 ☕ *– Coffee Break –*

11:45–12:30 ⇝ Thomas Guhr
Non-Stationarity in Financial Markets: Dynamics of Market States versus Generic Features

12:30–13:15 ⇝ Zdzisław Burda
Signal and noise in financial correlation matrices

13:15–14:30 *– Lunch Break –*

14:30–15:15 ⇝ Barbara Gentz
Small eigenvalues and mean transition times for irreversible diffusions

15:15–16:00 ⇝ Martin Venker
New Results on Local Universality of Repulsive Particle Systems

16:00–16:30 ☕ *– Coffee Break –*

16:40–17:55 *– Parallel Sessions –*

Notes

Venue. The main venue is the ZiF Plenarsaal .

Coffee Breaks. These take place in the ZiF Cafeteria.

Parallel Sessions on Monday Afternoon

Time	**Session 1** *T2-137*	**Session 2** *T2-208*	**Session 3** *T2-238*
16:40–17:05	Jesper R. Ipsen *Products of Independent Rectangular Gaussian Matrices*	Narges Rezvani *Entrance Laws of Generalized Ornstein-Uhlenbeck Processes*	Mykola Lebid *On fractal faithfulness and generalized infinite Bernoulli convolutions*
17:05–17:30	Alexey Naumov *Limit theorems for two classes of random matrices with dependent entries*	Diana Putan *A general uniqueness criterion for Gibbs measures with non-compact spins and some applications*	Janna Lierl *Geometric boundary Harnack principle on fractals*
17:30–17:55	Anna Reshetenko *Asymptotic Expansion in Classical and Free probability*	Viktor Bezborodov *Birth and death Markov evolutions in configuration spaces*	Yuhua Sun *Nonexistence results for solutions of semilinear inequality and systems on Riemannian manifolds*

Tuesday

9:00–9:45 ⇝ Yongsheng Song
Processes with stationary and independent increments under G expectation

9:45–10:30 ⇝ Shaolin Ji
A stochastic recursive optimal control problem under the G-expectation framework

10:30–11:15 ⇝ Xin Guo
Martingale Problems Under Nonlinear Expectations

11:15–11:45 ☕ *– Coffee Break –*

11:45–12:30 ⇝ Goran Peskir
Three-Dimensional Brownian Motion and the Golden Ratio Rule

12:30–13:15 ⇝ Patrick Cheridito
CoCos: pricing, hedging and design

13:15–14:30 *– Lunch Break –*

14:30–18:00 *– Excursion –*

Wednesday

9:00–9:45 ⇝ Gordan Žitković
Facelifting in mathematical finance

9:45–10:30 ⇝ Peter Bank
Superreplication when trading at market indifference prices

10:30–11:15 ⇝ Laurent Denis
An approach of uncertainty in finance based on capacity theory

11:15–11:45 ☕ *– Coffee Break –*

11:45–12:30 ⇝ Beatrice Acciaio
Robust Pricing and Hedging: from Mass Transport to Trajectorial Inequalities

12:30–13:15 ⇝ Agnès Sulem
A stochastic control approach to robust duality in utility maximization

13:15–14:30 *– Lunch Break –*

14:30–15:20 *– Parallel Sessions –*

15:20–16:00 ☕ *– Coffee Break –*

16:00–16:45 ⇝ Savaş Dayanik
Dynamic bidding strategies in search-based advertising

16:45–17:30 ⇝ Mihail Zervos
Irreversible capacity expansion with possible default

17:30–18:15 ⇝ Jan Obloj
On new advancements in Robust Pricing and Hedging

18:15– *– Conference Dinner –*

Parallel Sessions on Wednesday Afternoon

Time	**Session 1** *U2-216*	**Session 2** *U2-223*	**Session 3** *U2-228*
14:30–14:55	Xiangchan Zhu *Local existence and non-explosion of solutions for semilinear stochastic equations driven by multiplicative noise*	Giorgio Ferrari *On an Integral Equation for the Free-Boundary of Stochastic, Irreversible Investment Problems*	Sigrid Kallblad *Ambiguity averse portfolio optimization with respect to quasiconcave utility functionals*
14:55–15:20	Rongchan Zhu *Stochastic semilinear equations and their associated Fokker-Planck equations*	Patrick Beißner *Radner Equilibria under Volatility Uncertainty*	Thomas Löbbe *On the Probabilistic Behaviour of Multivariate Lacunary Systems*

Thursday

9:00–9:45 ⇝ Salvatore Federico
Characterization of the optimal boundaries in reversible investment problems

9:45–10:30 ⇝ Nathan Glatt-Holtz
Recent results on inviscid limits for the stochastic Navier-Stokes equations and related systems

10:30–11:15 ⇝ Benjamin Gess
Finite speed of propagation for stochastic porous media equations

11:15–11:45 ☕ *– Coffee Break –*

11:45–12:30 ⇝ Zechun Hu
Hunts Hypothesis

12:30–13:15 ⇝ Feng-Yu Wang
Equivalent Harnack and Gradient Inequalities for Pointwise Curvature Lower Bound

13:15–14:30 *– Lunch Break –*

14:30–15:15 ⇝ Shuanjian Tang
Optimal Switching of SDEs with Random Coefficients and Associated System of Reflected Backward Stochastic PDEs

15:15–16:00 ⇝ Jonas Tölle
Multi-valued stochastic evolution equations

16:00–16:30 ☕ *– Coffee Break –*

16:30–17:15 ⇝ Wilhelm Stannat
Stochastic stability of travelling waves in nerve axon equations

17:15–18:00 ⇝ Saïd Hamadene
Viscosity Solutions of Systems of Variational Inequalities with Interconnected Bilateral Obstacles

Friday

9:00–9:45 ⇝ Fuzhou Gong
Spectral Gaps of Schrödinger Operators and Diffusion Operators on Abstract Wiener Spaces

9:45–10:30 ⇝ Frederik Herzberg
Stochastic analysis with infinitesimals - a "radically elementary" approach

10:30–11:15 ⇝ Filippo Santambrogio
Mean Field Games and population dynamics under density constraints

11:15–11:45 ☕ *– Coffee Break –*

11:45–12:30 ⇝ Chuanzhong Chen
Generalized Feynman-Kac semigroups and stochastic calculus

12:30–13:15 ⇝ Giuseppe Da Prato
Maximal Sobolev regularity for a class of second order elliptic PDEs in infinite dimension

13:15–14:30 *– Lunch Break –*

14:30–15:15 ⇝ Francesco Russo
Stochastic calculus via regularization in Banach spaces and applications

15:15–16:00 ⇝ Sergio Albeverio
T.B.A.

16:00– *— Closing —*

Abstracts

(sorted alphabetically by the family names of the speakers)

Beatrice ACCIAIO

Robust Pricing and Hedging: from Mass Transport to Trajectorial Inequalities

We will discuss the advantages of relating the robust pricing problem to the theory of mass transportation. Mathematically the crucial difference is that transport plans are required to be martingales in our setting. In particular, we will see how the duality theorem from optimal transport can be used to establish new robust super-replication results. This dual viewpoint also provides new insights on classical martingale inequalities. For instance, we establish a (new) sharp version of the classical Doob maximal inequality.

Sergio ALBEVERIO

T.B.A.

Peter BANK

Superreplication when trading at market indifference prices

We consider a large investor who seeks to superreplicate a given contingent claim. Trading is done dynamically at market indifference prices as introduced in earlier joint work with D. Kramkov. The nonlinearities of this model for an illiquid financial market turn out to make this fundamental problem surprisingly intricate. It is possible (and economically nonetheless reasonable), for instance, that an asset can be replicated with two different initial levels of wealth without this creating arbitrage opportunities. We introduce a notion of efficient friction suitable for this price impact model and show how this ensures existence of optimal superrelicating portfolios under some assumptions on the market makers utility functions. We also establish efficient friction to hold when payoffs of traded assets are specified via Levy processes or certain affine processes, e.g., like in Barndorff-Nielsen-Shepard stochastic volatility models. This is joint work with Selim Gokay.

Patrick BEIßNER

Radner Equilibria under Volatility Uncertainty

The present paper considers a class of general equilibrium economies when the primitive uncertainty model features uncertainty about continuous-time volatility. This requires a set of mutually singular priors, which do not share the same zero sets. For this setting we introduce an appropriate commodity space and the dual of linear and continuous price systems. All agents in the economy are heterogeneous in their preference for uncertainty. Each utility functional is of variational type. The existence of the equilibrium is approached by a generalized excess utility fixed point argument. Such Arrow-Debreu allocations can be implemented into a Radner economy with continuous-time trading. Effectively completeness of the market spaces alters to a robust property. Only mean unambiguous claims satisfying equivalently the classical martingale representation property are elements of the marketed space.

Volker BETZ

Planar spatial random permutations

Spatial random permutations in two dimensions appear to be a rich an intriguing probabilistic model, with connections to mean curvature motion, the Bose gas, random fractal and possibly SLE. I will give an introduction into the model and explain some special cases where the connections above are evident. Furthermore, I will show numerical evidence for the validity of these connections also in more general situations.

Viktor BEZBORODOV

Birth and death Markov evolutions in configuration spaces

We introduce spatial birth and death process with finite number of points as the solution of a stochastic equation. We also consider some properties and asymptotic behavior of the process.

Zdzisław BURDA

Signal and noise in financial correlation matrices

Portfolio theory and random matrix theory were formulated in the fifties by Nobel Prize winners in economy (Harry Markowitz) and in physics (Eugene Wigner). According to portfolio theory the optimal asset allocation requires knowledge of the asset covariance matrix. This matrix is usually estimated using historical data. It contains statistical noise. The estimator is called sample covariance or historical covariance. In recent years many methods have been developed to find relations between the historical covariance matrix and the underlying genuine covariance matrix. This problem is sometimes

referred to as the problem of covariance cleaning. Methods of random matrix theory turned out to be very useful for cleaning sample covariance matrices. Nowadays these methods find applications in portfolio management and are used in practice by some hedge funds. We shortly discuss main ideas behind applications of random matrix theory to portfolio theory. In particular we show how to derive exact relations between eigenvalue distributions of the sample covariance and the genuine covariance matrix. The eigenvalues of the covariance matrix play the role of risk factors in the market.

Chuanzhong CHEN

Generalized Feynman-Kac semigroups and stochastic calculus

In this talk, we will present some results on the strong continuity of Generalized Feynman-Kac semigroups under the framework of Dirichlet forms. Also, we will state two different definitions of stochastic calculus under the framework of local semi-Dirichlet forms. Then, we will overview two methods on getting Fukushimas decomposition under the framework of semi-Dirichlet forms. Recently, Sun Wei and Zhang Jing get the representation of semi-Dirichlet form. Based on these new tools, it is hopeful to generalize the strong continuity of Generalized Feynman-Kac semigroups and stochastic calculus to more general case.

Patrick CHERIDITO

CoCos: pricing, hedging and design

After the financial crisis of 2007-2009 a number of financial institutions have issued contingent convertible bonds to protect their balance sheets in terms of crisis. They are typical hybrid products in that they depend on different sources of risk: interest rate risk, conversion risk and equity risk. This talk discusses the pricing and hedging of contingent convertibles as well as their design.

Giuseppe DA PRATO

Maximal Sobolev regularity for a class of second order elliptic PDEs in infinite dimension

We consider an elliptic Kolmogorov equation $\lambda u - Ku = f$ in an infinite dimensional separable Hilbert space H. The Kolmogorov operator K is associated to an infinite dimensional convex gradient system. Under mild assumptions we prove that for $\lambda > 0$ and $f \in L^2(X, \nu)$ the weak solution u belongs to $W^{2,2}(X, \nu)$, where ν is the log-concave probability measure of the system. Moreover maximal estimates on the gradient of u are proved. The maximal regularity results are applied to perturbed non gradient systems. The general results are applied to Kolmogorov equations associated to reaction–diffusion and Cahn–Hilliard stochastic PDE's.

The talk is based on the paper "G. Da Prato and A. Lunardi, arXiv:1208.0437".

Savaş DAYANIK

Dynamic bidding strategies in search-based advertising

Search-based advertising allows the advertisers to run special campaigns targeted to different groups of potential consumers at low costs. Google, Yahoo and Microsoft advertising programs allow the advertisers to bid for an ad position on the result page of a user when the user searches for a keyword that the advertiser relates to its products or services. The expected revenue generated by the ad depends on the ad position, and the ad positions of the advertisers are concurrently determined after an instantaneous auction based on the bids of the advertisers. The advertisers are charged only when their ads are clicked by the users. Each advertiser reserves a fixed finite daily budget, and the ads are not shown in the remainder of the day when the budget is depleted. Arrival times of keyword-search instances, ad positions, ad selections, and sales generated by the ads are random. Therefore, an advertiser faces a dynamic stochastic total net revenue optimization problem subject to a strict budget constraint. We formulate and solve this problem using stochastic dynamic programming. We illustrate the solution method on numerical examples. (This is joint work with M. Parlar, McMaster University, Canada)

Laurent DENIS

An approach of uncertainty in finance based on capacity theory

The purpose of this talk is to explain how to take into account, from a mathematical point of view, the model uncertainty in finance. The model uncertainty is specified by a very general set of laws which represents all the possible laws of the underlying assets. For example, the usual approach of utility maximization due to Von Neumann, Morgenstern provides conditions on an investor's preferences which guarantee that the utility of a terminal wealth can be expressed as follows:

$$u(X) = E_Q[U(X)]$$

where Q is a given probability and U is a utility function. Then, one has to maximize u over a set of admissible strategies.
However, this paradigm of expected utility has clearly some deficiencies : it is not satisfactory in dealing with model uncertainty as predicted by the famous Ellsberg paradox. In order to emphasize the importance of considering model uncertainty, one can argue that there are lots of probability models in mathematical finance to describe price of assets but none of them really succeeds in describing the market. For example, in the Black Scholes model, the volatility failed to be constant. So, it is difficult to choose a model but even when it is done, one has to deal with calibration of the parameters. For all these reasons it seems too restrictive to assume that the agents of the economy have a perfect knowledge of the probability law that governs the market. Therefore, in many papers, authors take into account model uncertainty, which is also called sometimes

Knightian uncertainty or model ambiguity. So, in order to consider model uncertainty in the problem of maximization of expected utility, Gilboa and Schmeidler introduced a robust version of the expected utility of the form

$$\inf_{Q\in\mathcal{P}} E_Q[U(X)]$$

where the infimum is taken over a whole class of possible probabilistic views of the given set of scenarios. Now, the aim of an investor is to maximize the robust utility function over the set of possible gains arising from admissible trading strategies.
There is a huge literature about this subject but the authors assume that all the probabilities in $\mathcal{P}$ are absolutely continuous with respect to a given reference measure P. It is known that this condition on $\mathcal{P}$ is violated in many examples.
The most famous example is the uncertain volatility model (UVM in short) introduced by Avellaneda et als. and Lyons which takes into account the difficulty of calibration of the volatility in the Black-Scholes model. In these works, authors used stochastic control techniques to price European options. Then, in [1] a framework is introduced in a more general case, encompassing the case of the UVM model, which permits to evaluate the cheapest super-replication price of a European contingent claim under model uncertainty. The main point is that the set of probabilities $\mathcal{P}$ is not necessarily dominated so it generates some new difficulties. The negligible sets are more difficult to define. When one works with a set of probability measures absolutely continuous with respect to a probability measure P, then a negligible set under P is negligible under each probability measure of the set of probabilities we consider. It is not true anymore when one works with a non dominated set of probability measures and in this case, one must consider what happens under each probability of the set.
In this talk, we shall present different approaches which permit to solve this problem: one based on the capacity theory and the other on the G-expectation and then make the link between them.

References

[1] Denis L. et Martini C., A theoretical framework for the pricing of contingent claims in the presence of model uncertainty , Annals of Applied Probability, Vol. 16.2, pp 827-852 (2006)

[2] Denis L., Hu Y et Peng S. , Function spaces and capacity related to a Sublinear Expectation: application to G-Brownian Motion Paths, Potential Analysis, Vol. 34.2 139-161 (2011).

Salvatore FEDERICO

Characterization of the optimal boundaries in reversible investment problems

This paper studies a reversible investment problem where a social planner aims to control its capacity production in order to fit optimally the random demand of a good. Our model allows for general diffusion dynamics on the demand as well as general cost

functional. The resulting optimization problem leads to a degenerate two-dimensional singular stochastic control problem, for which explicit solution is not available in general and the standard verification approach can not be applied a priori. We use a direct viscosity solutions approach for deriving some features of the optimal free boundary function, and for displaying the structure of the solution. In the quadratic cost case, we are able to prove a smooth-fit C_2 property, which gives rise to a full characterization of the optimal boundaries and value function.

Giorgio FERRARI

On an Integral Equation for the Free-Boundary of Stochastic, Irreversible Investment Problems

In this paper we derive a new handy integral equation for the free- boundary of infinite time horizon, continuous time, stochastic, irreversible investment problems with uncertainty modeled as a one-dimensional, regular diffusion X. The new integral equation allows to explicitly find the free- boundary $b(.)$ in some so far unsolved cases, as when the operating profit function is not-separable and X is a three-dimensional Bessel process or a CEV process. Our result follows from purely probabilistic arguments. Indeed, we first show that $b(X(t)) = l^*(t)$, with $l^*(t)$ the unique optional solution of a representation problem in the spirit of Bank-El Karoui [1]; then, thanks to such identification and the fact that l^* uniquely solves a backward stochastic equation, we find the integral problem for the free-boundary.

References

[1] P. Bank, N. El Karoui A Stochastic Representation Theorem with Applications to Optimizazion and Obstacle Problems. The Annals of Probability 32 (2004), pp. 10301067.

Barbara GENTZ

Small eigenvalues and mean transition times for irreversible diffusions

The spectral theory of reversible diffusions in the small-noise limit is well understood. The small eigenvalues of the generator have been analyzed by a number of different methods, including large deviations, semiclassical analysis and potential theory. The study of the irreversible case, which involves a non-selfadjoint generator, is substantially more difficult. We will discuss an approach based on Laplace transforms of hitting times for Markov chains with continuous state space. These Markov chains arise from random Poincaré maps.
The proposed approach provides information on the exponentially small eigenvalues of the generator, and on mean transition times between attractors. As an illustration, we will present a detailed analysis of the asymptotic behaviour of the first-passage time of a planar diffusion through an unstable periodic orbit in the small-noise limit.
Joint work with Nils Berglund (Orléans).

Benjamin GESS

Finite speed of propagation for stochastic porous media equations

We prove finite speed of propagation for stochastic porous media equations perturbed by linear multiplicative space-time rough signals. Explicit and optimal estimates for the speed of propagation are given. The estimates are then used to prove that the corresponding random attractor has infinite fractal dimension.

Nathan GLATT-HOLTZ

Recent results on inviscid limits for the stochastic Navier-Stokes equations and related systems

One of the original motivations for the development of stochastic partial differential equations traces it's origins to the study of turbulence. In particular, invariant measures provide a canonical mathematical object connecting the basic equations of fluid dynamics to the statistical properties of turbulent flows. In this talk we discuss some recent results concerning inviscid limits in this class of measures for the stochastic Navier-Stokes equations and other related systems arising in geophysical and numerical settings. This is joint work with Peter Constantin, Vladimir Sverak and Vlad Vicol.

Thomas GUHR

Non-Stationarity in Financial Markets: Dynamics of Market States versus Generic Features

Non-stationarity is a fundamental feature of many complex systems. In financial markets, the fluctuations of volatilities and correlations are typical signatures. By detailed data analysis, we show that distinct market states exists which evolve dynamically, i.e. the market jumps back and forth, while the states emerge and disappear. At first sight paradoxically, the very same non-stationarity also produces generic features. We uncover them by analyzing return distributions and set up a random matrix model to describe them.

Xin GUO

Martingale Problems Under Non-linear Expectations

We consider the martingale problem under the framework of nonlinear expectations, analogous to that in a probability space in the seminal paper of Stroock and Varadhan (1969). We first establish an appropriate comparison theorem and the existence result for the associated state-dependent fully non-linear parabolic PDEs. We then construct the conditional expectation from the viscosity solution of the PDEs, and solve the existence of martingale problems. Under this non-linear expectation space, we further

develop the stochastic integral and the Itô's type formula, which are consistent with Peng's *G*-framework. As an application, we introduce the notion of weak solution of SDE under the non-linear expectation.
This is a joint work with C. Pan and S. G Peng.

Saïd HAMADENE

Viscosity Solutions of Systems of Variational Inequalities with Interconnected Bilateral Obstacles

The talk is related to existence and uniqueness of a solution in viscosity sense of a system of PDEs with bilateral inter-connected obstacles. This problem is connected with two players zero-sum switching game. Mainly, under the non-free loop property on the switching costs and monotonicity of the generators, we show existence and uniqueness of a solution for the system. Our method combines probabilistic tools and analytical ones as well. It is based on systems of reflected backward stochastic differential equations and Perron's method.

Frederik HERZBERG

Stochastic analysis with infinitesimals - a "radically elementary" approach

In his monograph "Radically Elementary Probability Theory" [Annals of Mathematics Studies, vol. 117 (1987)], Edward Nelson has laid the foundations of a theory of continuous-time stochastic processes that is accessible to "anyone who can add, multiply, and reason"; for this purpose, Nelson dispenses of measure theory and instead employs a tiny fragment of nonstandard analysis. This talk will summarize a research program that continues Nelson's work by developping a 'radically elementary' approach to stochastic analysis and its applications, especially in financial mathematics and quantum mechanics. A major part of the results has recently appeared in print [F.H., "Stochastic analysis with infinitesimals", Lecture Notes in Mathematics, vol. 2067 (2013)].

Fuzhou GONG

Spectral Gaps of Schrödinger Operators and Diffusion Operators on Abstract Wiener Spaces

How to use the information of coefficients in partial differential operators to get the information of spectrum of the operators? There exists a long literature of studying this problem from theory of diffusion processes and partial differential equations, and there are a lot of interesting problems need to answer. Among them there is a fundamental gap conjecture observed by Michiel van den Berg [J. Statist. Phys. 31(1983),no.3,623-637] and was independently suggested by Ashbaugh and Benguria [Proc. Amer. Math. Soc.

105(1989),no.2,419-424] and Yau [Nonlinear analysis in geometry, Monographies de L'Enseignement Mathématique,Vol.33,L'Enseignement Mathématique,Geneva, 1986. Série des Conférences de l'Union Mathématique Internationale,8], which gave an optimal lower bound of $\lambda_1 - \lambda_0$, the distance between the first two Dirichlet eigenvalues of a Schrödinger operator $-\Delta + V$ on a bounded uniformly convex domain Ω with a weakly convex potential V . By introducing the notion of modulus of convexity for functions, and studying the relationship between the modulus of convexity for V and the modulus of log-concavity for the first eigenfunction (i.e. ground state) of Schrödinger operator $-\Delta + V$ through that of the one dimensional corresponding problems, Andrews and Clutterbuck [J. Amer. Math. Soc. 24 (2011), no. 3, 899916] recently solved the fundamental gap conjecture. More interestingly, they proved a fundamental gap comparison theorem, that compare the fundamental gap of the Schr dinger operator $-\Delta + V$ with that of the one dimensional corresponding operator.

In this talk we extend the fundamental gap comparison theorem of Andrews and Clutterbuck to the infinite dimensional setting. More precisely, we proved that the fundamental gap of the Schrödinger operator $-\mathcal{L}_* + V$ ($\mathcal{L}_*$ is the OrnsteinUhlenbeck operator) on the abstract Wiener space is greater than that of the one dimensional operator $-\frac{d^2}{ds^2} + s\frac{d}{ds} + \tilde{V}(s)$, provided that $\tilde{V}$ is a modulus of convexity for V . Similar result is established for the diffusion operator$-\mathcal{L}_* + \nabla F.\nabla$. The main results are as follows. Let (W, H, μ) be an abstract Wiener space and $\mathcal{L}_*$ the OrnsteinUhlenbeck operator on W associated to the symmetric Dirichlet form $\mathcal{E}_*(f, f) = (f, -\mathcal{L}_* f)$ with domain $\mathcal{D}[\mathcal{E}_*] = D_1^2(W, \mu)$ (i.e. $f \in L^2(W, \mu)$ with its Malliavin derivative $\nabla f \in L^2(W, H)$). Let $V \in D_1^2(W, \mu)$ for some $p > 1$ be a potential satisfying the *KLMN condition*, then one can define $\mathcal{L} = \mathcal{L} + V$ to be a self-adjoint Schrödinger operator bounded from below.

Correspondingly, let $\mathcal{L} = \frac{d^2}{ds^2} - s\frac{d}{ds}$ be the one-dimensional OrnsteinUhlenbeck operator on R^1 with respect to the Gaussian measure $d\gamma_1 = (4\pi)^2 exp(\frac{s^2}{4})ds$. Let $\tilde{V} \in C^1(R^1) \cap L^1(R^1, \gamma_1)$ be a symmetric potential satisfying the *KLMN condition* too. Then $\tilde{\mathcal{L}} = \tilde{\mathcal{L}} + \tilde{V}$ is bounded from below. For convenience, a tilde will be added to all notations relative to $\tilde{\mathcal{L}}$ and $\tilde{V}$. Let$< ., . >_H$ denote the inner product in the Camero-Martin space H, and $|.|_H$ the norm.

Theorem A: Suppose for almost all $w \in W$ and every $h \in H$ with $h \neq 0$,

$$\left\langle \nabla V(w+h) - \nabla V(w), \frac{h}{|h|_H} \right\rangle_H \geqslant 2\tilde{V}'\left(\frac{|h|_H}{2}\right)$$

Then there exists a comparison

$$\lambda_1 - \lambda_0 \geqslant \tilde{\lambda}_1 - \tilde{\lambda}_0.$$

Hence, the existence of the spectral gap of $\mathcal{L}$ on Wiener space can sometimes be reduced to one dimensional case. According to Andrews and Clutterbucks notion, $\tilde{V}$ is a modulus of convexity for V . However, V doesnt need to be convex at all. The next result gives the modulus of log-concavity for the ground state ϕ_0 of $\mathcal{L}$.

Theorem B: Assume the same condition as in Theorem A and the gap $\tilde{\lambda}_1 - \tilde{\lambda}_0 > 0$. Then $-\mathcal{L}$ and $-\tilde{\mathcal{L}}$ have a unique ground state ϕ_0 and $\tilde{\phi}_0$ respectively. Moreover, for

almost all $w \in W$ and every $h \in H$ with $h \neq 0$,

$$\left\langle \nabla \log \phi_0(w+h) - \nabla \log \phi_0(w), \frac{h}{|h|_H} \right\rangle_H \leqslant (\log \tilde{\phi}_0)' \left(\frac{|h|_H}{2} \right).$$

We also consider the diffusion operator $\mathcal{L} = -\mathcal{L} + \nabla F.\nabla$ on the Wiener space and we want to compare its spectral gap with the one dimensional operator $-\tilde{\mathcal{L}} = -\frac{d^2}{ds^2} + (s + \omega'(s))\frac{d}{ds}$. Although this kind of diffusion operator can be transformed to the Schrödinger type operator and their spectrum coincide with each other, the expression for the potential function V is a little complicated, hence it seems inappropriate to derive the gap comparison of diffusion operators from that of the transformed Schrödinger type operators. We shall directly establish the comparison theorem for spectral gaps of diffusion operators, and the main result is as follows.

Theorem C: Assume that $F \in D_1^p(W, R^1)$ satisfies $\int_W e^{-F} d\mu = 1$ and two functions F and ω are related by the following inequality: for all $h \in H$ and μ-a.e. $w \in W$,

$$\left\langle \nabla F(w+h) - \nabla F(w), \frac{h}{|h|_H} \right\rangle_H \geqslant 2\omega' \left(\frac{|h|_H}{2} \right).$$

Suppose also that $\omega \in C^1(R^1)$ is even, satisfying $\int_{R^1} e^{-\omega} d\gamma_1 = 1$ and $\lim_{t\to\infty}(\omega'(t) + t) = +\infty$. Then we have

$$\lambda_1 \geqslant \tilde{\lambda}_1.$$

Furthermore, the probabilistic proofs of fundamental gap conjecture were given by Fuzhou Gong and Dejun Luo most recently via the coupling by reflection of the diffusion processes.

Zechun HU

Hunts Hypothesis (H)

This talk contains three parts. In the first part, I will introduce the meaning of Hunts hypothesis (H) and several related equivalent potential principles. In the second part, I will introduce Getoors conjecture about Hunts hypothesis (H) for Levy processes and the existing results. In the final part, I will present some recent progresses on Getoors conjecture obtained by us. The talk is based on joint work with Wei Sun.

Jesper R. IPSEN

Products of Independent Rectangular Gaussian Matrices

Products of independent rectangular Gaussian matrices have applications in such diverge topics as quantum chromodynamics with finite chemical potential and concatenated scattering in wireless telecommunication. Here we discuss the correlation functions for the eigenvalues as well as the singular values of such product matrices, and

obtain explicit expressions for the correlations functions, which generalize the classical results for the (induced) Ginibre ensembles as well as the Wishart-Laguerre unitary ensemble.

Shaolin JI

A stochastic recursive optimal control problem under the G-expectation framework

In this paper, we study a stochastic recursive optimal control problem in which the objective functional is described by the solution of a backward stochastic differential equation driven by G-Brownian motion. Under standard assumptions, we establish the dynamic programming principle and the related Hamilton-Jacobi-Bellman (HJB) equation in the framework of G-expectation. Finally, we show that the value function is the viscosity solution of the obtained HJB equation.

Sigrid KALLBLAD

Ambiguity averse portfolio optimization with respect to quasiconcave utility functionals

While recent developments in [1] motivates the specification of risk- and ambiguity-averse preferences in terms of quasiconcave utility functionals, robust representations of the latter have been established in [2] and [3]. In this paper, we study the associated investment problem for an investor with a fixed finite horizon and who trades in a continuous-time stochastic market model. Similarly to what was done for concave monetary utility functionals (variational preferences) in [4], we prove existence of an optimal strategy and establish certain duality results by building on the classical results by Kramkov and Schachermayer (1999, 2003).

References

[1] Cerreia-Vioglio, S., Maccheroni, F., Marinacci, M. and L. Montrucchio: Uncertainty averse preferences: Journal of Economic Theory, 146, 1275-1330 (2011).

[2] Cerreia-Vioglio, S., Maccheroni, F., Marinacci, M. and L. Montrucchio: Complete monotone quasiconcave duality: Mathematics of Operations research, 36, 321-339 (2011).

[3] Drapeau, S. and M. Kupper: Risk preferences and their robust representation, Mathematics of Operations Research, 38, 28-62 (2013).

[4] Schied, A.: Optimal investments for risk- and ambiguity-averse preferences: a duality approach, Finance and Stochastics, 11, 107-129 (2007).

Mykola LEBID

On fractal faithfulness and generalized infinite Bernoulli convolutions

The talk will be devoted to the analysis of families Φ of coverings which are faithful for the Hausdorff dimension calculation on a given set E (i. e., special relatively narrow families of coverings leading to the classical Hausdorff dimension of an arbitrary subset of E) and which are natural generalizations of comparable net-coverings. They are shown to be very useful for the determination or estimation of the Hausdorff dimension of sets and probability measures. Some applications to the investigation of generalized infinite Bernoulli convolutions will be discussed.

References

[1] S. Albeverio, G. Torbin, *On fine fractal properties of generalized infinite Bernoulli convolutions*, Bull. Sci. Math., **132**(2008), P. 711-727.

[2] L., G. Torbin, *On generalized infinite Bernoulli convolutions with overlaps and their fractal properties*, Prob. Theory and Math. Statistics, **87**(2012), P. 89 - 104.

[3] S. Albeverio, G. Ivanenko, L., G. Torbin, *On the Hausdorff dimension faithfulness for covering families and its applications*, in press (available at http://arxiv.org/abs/math/0724662).

Janna LIERL

Geometric boundary Harnack principle on fractals

The boundary Harnack principle states that the ratio of any two functions that are positive and harmonic on a domain, is bounded near some part of the boundary where both functions vanish. A given domain may or may not have such a property, depending on the geometry of its boundary and the underlying metric space. Aikawa, Ancona, Gyrya and Saloff-Coste proved a scale-invariant boundary Harnack principle for domains in Euclidean space, and in non-fractal Dirichlet spaces, respectively, assuming that the domain is inner uniform. This is a large class of domains, which can have very rough boundary, but no cusps. I will present a scale-invariant boundary Harnack principle for inner uniform domains in metric measure Dirichlet spaces that satisfy a parabolic Harnack inequality. The result applies to fractal spaces, e.g. Sierpinski gasket. Applications of the boundary Harnack principle include two-sided bounds on the Dirichlet heat kernel, or the identification of the Martin boundary and the topological boundary of bounded inner uniform domains.

Thomas LÖBBE

On the Probabilistic Behaviour of Multivariate Lacunary Systems

Lacunary function systems of type $f(M_n x)$ for sequences of fast-growing non-singular $d \times d$-Matrices M_n and periodic functions f exhibit many properties of independent random variables like satisfying the Central Limit Theorem or the Law of the Iterated Logarithm. It is well-known that this behaviour sensitively depends on number theoretic properties of M_n as well as analytic properties of f. Classical techniques which are used in the one-dimensional case are essentially based on Fourier analysis making it almost impossible to use a similar approach in the multivariate setting. In 2010 Aistleitner and Berkes introduced a new method proving the Central Limit Theorem in the one-dimensional case by approximating f by piecewise constant periodic functions which form a martingale differences sequence and using a Berry-Esseen type inequality. Later this approach was used to show the Law of the Iterated Logarithm by a consequence of Strassen's almost sure invariance principle. In this talk I develop this method to prove the Central Limit Theorem and the Law of the Iterated Logarithm in the multidimensional case. Furthermore I apply these results to discrepancy theory.

Shunlong LUO

Measuring Quantum Non-Markovianity

In the study of open quantum systems, memory effects are usually ignored, and this leads to dynamical semi-groups and Markovian dynamics. However, in practice, non-Markovian dynamics is the rule rather than exception. With the recent emergence of quantum information theory, there is a flurry of investigations of non-Markovian dynamics. In this talk, we first review several significant measures for non-Markovianity, such as deviation from divisibility, information exchange between a system and its environment, or entanglement with the environment. Then by exploiting the correlations flow between a system and an arbitrary ancillary, we study a considerably intuitive measure for non-Markovianity by use of correlations as quantified by the quantum mutual information rather than entanglement. The measure captures quite directly and deeply the characteristics of non-Markovianity from the perspective of information. A simplified version based on Jamiolkowski-Choi isomorphism which encodes operations via bipartite states and does not involve any optimization is also proposed.

Zhi-Ming MA

Markov jump processes in modeling coalescent with recombination

The stochastic evolution of a DNA segment that experiences recombination is a complex process, there have been some analyses based on simulations or using heuristic methods. In this talk I shall propose a rigorous mathematical model describing coalescent with recombination. Our model employs heavily the theory of Markov jump

processes. The new model provides a unified interpretation for the algorithms of simulating coalescent with recombination, and will facilitate the study on recombination and advance our understanding of recombination. The talk is based on our recent results in collaboration with researchers in the Institute of Applied Mathematics of CAS,the Partner Institute for Computational Biology of CAS, and Beijing Jiaotong University.

Alexey NAUMOV

Limit theorems for two classes of random matrices with dependent entries.

In this talk we consider ensembles of random symmetric matrices with random field type dependence. Suppose that the entries of the matrix have zero mean and finite variances which can be different numbers. Assuming that the average of the normalized sums of variances in each row converges to one and Lindeberg condition holds true we prove that the empirical spectral distribution of eigenvalues converges to Wigners semicircle law. We also provide analogue of this result for sample covariance matrices and prove convergence of the empirical spectral distribution of eigenvalues to the Marchenko-Pastur law. This talk is based on the joint works of F. Goetze, A. Naumov and A. Tikhomirov.

Jan OBLOJ

On new advancements in Robust Pricing and Hedging

In the classical approach in mathematical finance one starts by postulating a probabilistic model and computes prices and hedging strategies implied by this model. In contrast, in the robust (model-free) approach, going back to the seminar work of Hobson (1998), one starts with market prices of vanilla options and determines the range of no-arbitrage prices for an exotic option together with super- and sub- hedging strategies which enforce the price bounds.

I present several new developments which enrich the classical robust setting. First, I consider the case when prices for options with n intermediate maturities are traded. I discuss both general no pricing-hedging duality gap results as well as explicit solutions for lookback options.

Secondly, building on the framework in Cox and Obloj (2011), I consider the case when time series information is used to further constraint the set of possible paths of risky assets. This essentially corresponds to limiting the support of compatible probabilistic models. I formulate Robust Market Models, i.e. a robust approach to the market models of e.g. Carmona and Nadtochiy (2009) or Schweizer and Wissel (2008). Focusing on barrier options, I show that this approach is capable of interpolating between the model-independent setting (recovering Brown, Hobson and Rogers (2001)) and a model-specific setting (recovering static hedging of Carr and Nadtochiy (2011)).

Based on joint works with Pierre Henry-Labordere, Zhaoxu Hou, Sergey Nadtochiy, Peter Spoida and Nizar Touzi.

Goran PESKIR

Three-Dimensional Brownian Motion and the Golden Ratio Rule

We show that the first time at which the excursion of the radial part of three-dimensional Brownian motion away from its running minimum and the running minimum itself form the golden ratio is as close as possible to the time of the ultimate minimum in a normalised mean deviation sense. Among other things this offers a rigourous optimality argument for the choice of the golden retracement in technical analysis of asset prices.
Joint work with K. Glover and H. Hulley

Diana PUTAN

A general uniqueness criterion for Gibbs measures with non-compact spins and some applications

An improved version of the Dobrushin-Pechersky uniqueness criterion for Gibbs measures will be presented. This result was so far poorly recognized, however we are able not only to give a clear proof for it, but also to present its applicability for certain models of interacting particle systems where the classical Dobrushin criterion does not work.

Anna RESHETENKO

Asymptotic Expansions in Classical and Free probability

In this talk we discuss asymptotic expansion in classical and free probability. In a class of classical limit theorems asymptotic expansions can be obtained by a sequence of functions, which are smooth, symmetric, compatible and have vanishing first derivatives at zero. In free probability, where a new type of (nonlinear) convolution for spectral distributions is used, the same scheme can be applied. An asymptotic expansion for the local free central limit theorem is shown via this method.

Narges REZVANI

Entrance Laws of Generalized Ornstein-Uhlenbeck Processes

This talk is devoted to the analysis of the set of all evolution systems of measures and to showing the existence of a representation for an arbitrary evolution system of measures as a convex combination of their extremal points. Then, the extremal points of this set is completely characterized for the particular case of generalized Mehler semigroups which are the transition semigroup of generalized time-inhomogeneous Ornstein- Uhlenbeck processes.

Francesco RUSSO

Stochastic calculus via regularization in Banach spaces and applications

This talk is based on collaborations with Cristina Di Girolami (Univ. Le Mans) and Giorgio Fabbri (Univ. Evry).

Finite dimensional calculus via regularization was first introduced by the speaker and P. Vallois in 1991. One major tool in the framework of that calculus is the notion of covariation $[X, Y]$ (resp. quadratic variation $[X]$) of two real processes X, Y (resp. of a real process X). If $[X]$ exists, X is called finite quadratic variation process. Of course when X and Y are semimartingales then $[X, Y]$ is the classical square bracket. However, also many real non-semimartingales have that property. Particular cases are Föllmer-Dirichlet and weak Dirichlet processes, introduced by M. Errami, F. Gozzi and the speaker. Let $(\mathcal{F}_t, t \in [0, T])$ be a fixed filtration. A weak Dirichlet process is the sum of a local martingale M plus a process A such that $[A, N] = 0$ with respect to all the local martingales related to the given filtration.
The lecture presents the extension of that theory to the case when the integrator process takes values in a Banach space B. In that case very few processes have a finite quadratic variation in the classical sense of Métivier-Pellaumail. An original concept of quadratic variation (or χ-quadratic variation) is introduced, where χ is a subspace of the dual of the projective tensor product $B\hat{\otimes}_\pi B$. Two main applications are considered.

- Case $B = C([-T, 0])$. One can express a Clark-Ocone representation formula of a path-dependent random variable with respect to an underlying which is a non-semimartingale withe finite quadratic variation. The representation is linked to the solution of an infinite dimensional PDE on $[0, T] \times B$.
- Case when B is a separable Hilbert space H. One investigates quadratic variations of processes which are solutions of an evolution equation, typically a mild solution of SPDEs.

References

[1] C. Di Girolami, F. Russo (2011). *Clark-Ocone type formula for non-semimartingales with non-trivial quadratic variation*. Comptes Rendus de l'Académie des Sciences, Section Mathématiques. Number 3-4, pp 209–214. Vol. 349.

[2] C. Di Girolami and F. Russo (2011). *Generalized covariation and extended Fukushima decompositions for Banach space valued processes. Application to windows of Dirichlet processes.* Preprint HAL-INRIA http://hal.inria.fr/inria-00594871/fr/
To appear: Infinite Dimensional Analysis, Quantum Probability and Related Topics (IDAQP).

[3] R. Coviello, C. Di Girolami, F. Russo (2011). *Stochastic calculus related financial assets without semimartingales.* Bulletin Sciences Mathématiques, vol. 135, pp. 733–774.

[4] C. Di Girolami, F. Russo (2010). *Generalized covariation for Banach valued processes and Itô formula*. To appear: Osaka Journal of Mathematics. *http://hal.archives-ouvertes.fr/inria-00484993/fr/*.

[5] G. Fabbri, F. Russo (2012). Infinite dimensional weak Dirichlet processes, stochastic PDEs and optimal control. HAL-INRIA, Preprint. *http://hal.inria.fr/hal-00720490*

Filippo SANTAMBROGIO

Mean Field Games and population dynamics under density constraints

I will start from a microscopic formulation for crowd movement given by B. Maury and collaborators, where individual are small disks subject to non-overlapping constraints. This has later been turned into a continuous model where the key point is the constraint $\rho \leqslant 1$ and the fact that the velocity field advecting ρ in the continuity equation is the projection of the desired one on the set of admissible fields (where admissible fields are those that have negative divergence on the set where $\rho = 1$). The corresponding PDE is quite hard and existence has been proven via optimal transport techniques. This passes through a catching-up technique: first we move as if the constraint was not taken into consideration, then we project on the set of admissible densities. After that, I'll introduce the (nowadays classical) mean-field game models with density penalization and then provide ideas to replace costs depending on the density $\rho(x)$ with constraints. The main difficulty is making it meaningful: indeed, if we only impose the constraint $\rho \leqslant 1$, then every agent, facing an already admissible density ρ, may move with no special constraints, since one only extra agent will never violate the constraint. In order to give a proper definition of the equilibrium and of the equations, we will use the pressure field. Finally, both models (crowd motion without equilibrium issues and MFG) will be made stochastic, by inserting a diffusion term, and I will suggest some catching-up methods to produce solutions to the relevant equations.

Yongsheng SONG

Processes with stationary and independent increments under G expectation

Our purpose is to investigate properties for processes with stationary and independent increments under G-expectation. As applications, we prove the martingale characterization of G-Brownian motion and present a pathwise decomposition theorem for generalized G-Brownian motion.

Wilhelm STANNAT

Stochastic stability of travelling waves in nerve axon equations

We study stochastic partial differential equations modelling the propagation of the action potential along the nerve axon of a single neuron subject to channel noise fluctuations, including stochastic FitzHugh-Nagumo systems. Stochastic stability of the action potential is proven using functional inequalities and an implicitly defined phase adaption. Our approach is new even for the deterministic case. A stochastic differential equation for the speed of the action potential is derived that allows to decompose the stochastic dynamics into the propagating action potential and noise fluctuations. Our approach also allows to calculate the probability of a propagation failure w.r.t. the to underlying channel noise fluctuations.

References

[1] Wilhelm Stannat, Stability of travelling waves in stochastic Nagumo equations, arxiv:1301.6378.v1

Agnès SULEM

A stochastic control approach to robust duality in utility maximization

A celebrated financial application of convex duality theory gives an explicit relation between the following two quantities:

1. The optimal terminal wealth $X^*(T) := X_{\varphi^*}(T)$ of the classical problem to maximize the expected U-utility of the terminal wealth $X_\varphi(T)$ generated by admissible portfolios $\varphi(t); 0 \leqslant t \leqslant T$ in a market with the risky asset price process modeled as a semimartingale

2. The optimal scenario $\frac{dQ^*}{dP}$ of the dual problem to minimize the expected V-value of $\frac{dQ}{dP}$ over a family of equivalent local martingale measures Q. Here V is the convex dual function of the concave function U.

In this paper we consider markets modeled by Itô-Lévy processes, and in the first part we extend the above result in this setting, based on the maximum principle in stochastic control theory. We prove in particular that the optimal adjoint process for the robust primal problem coincides with the optimal density process, and that the optimal adjoint process for the robust dual problem coincides with the optimal wealth process. An advantage with our approach is that it also gives an explicit relation between the optimal portfolio φ^* and the optimal measure Q^*, in terms of backward stochastic differential equations. We also obtain that the existence of an optimal scenario is equivalent to the replicability of a related T-claim.
In the second part we present robust (model uncertainty) versions of the optimization problems in (i) and (ii), and we prove a relation between them. In particular, we show

explicitly how to get from the solution of one of the problems to the solution of the other. We illustrate the results with explicit examples.

Yuhua SUN

Nonexistence results for solutions of semilinear inequality and systems on Riemannian manifolds

We investigate the nonexistence of non-negative solution to the following differential inequality

$$div(A(x)\nabla u) + V(x)u^{\sigma} \leqslant 0, \tag{1}$$

on a complete Riemannian manifold, where $\sigma \geqslant 0$ is a parameter. We obtain that, when $\sigma > 1$, if the following inequalities, for some reference point x_0

$$c(1 + d(x, x_0))^{-\delta_1} \leqslant \frac{V}{||A||} \leqslant C(1 + d(x, x_0))^{\delta_2}, \tag{*}$$

and

$$v(B(x_0, r)) \leqslant Cr^p \ln^q r, \tag{vol}$$

hold for all large enough r. Here δ_1, δ_2 are arbitrary non-negative values, $B(x_0, r)$ is a geodesic ball. Then the only non-negative solution of (1) is identically equal to zero.

For the case $0 \leqslant \sigma \leqslant 1$, if $\delta_1 < 2$ in (*), we also has the same result. We also construct some example to show that there may exist solution to (1) when $\sigma = 1$ and $\delta_1 = 2$.

Furthermore, we also investigate semilinear system for $\sigma_1, \sigma_2 > 1$

$$\begin{cases} div(A(x)\nabla u) + V(x)v^{\sigma_1} \leqslant 0, \\ div(A(x)\nabla v) + V(x)u^{\sigma_2} \leqslant 0. \end{cases} \tag{2}$$

we find that assume (*) holds, and either (i.)$\sigma_1 \neq \sigma_2, \delta_2 + \delta_1\sigma_1\sigma_2 < 2(\sigma_1\sigma_2 - 1)$ or (ii.)$\sigma_1 = \sigma_2$ holds. Under the volume estimate (vol), then any non-negative solution of (2) is identically equal to 0.

Shuanjian TANG

Optimal Switching of SDEs with Random Coefficients and Associated System of Reflected Backward Stochastic PDEs

In this talk, we are concerned with optimal switching of stochastic differential equations with random coefficients. The associated HJB equation turns out to be a system of backward stochastic partial differential equations (BSPDEs) with oblique reflection. The existence of the adapted strong solution is obtained by the penalization method, the monotone convergence, and the a priori estimations. The uniqueness is obtained by a verification method (the first component of any adapted solution is shown to be the vector value of a switching problem for BSDEs).

Jonas TÖLLE

Multi-valued stochastic evolution equations

Multi-valued stochastic (partial) differential equations, in short MSDE (MSPDE), have gathered some attention in the last few years. For them, several different approaches have been introduced in order to construct unique solutions as well as associated stochastic flows (in the wider sense). Speaking in terms of abstract time evolutions, one would like to consider the (formal) MSDE (in a separable Hilbert space H)

$$\begin{cases} du(t) \in -A(u(t))\,dt + B(u(t))\,dg(t), & t \in (0,T] \\ u(0) = u_0, \end{cases} \tag{1}$$

where $A : H \to 2^H$ is a (maximal) monotone, multi-valued operator, $u_0 \in H$, $g : [0,T] \to H$ is a path of a càdlàg stochastic process admitting some reasonable stochastic differential dg, and B is a continuous linear-operator-valued map. Because of the generic path-irregularity of $\frac{d}{dt}g$, we can neither employ the classical (time-dependent) theory via Yosida-regularization, nor the nonlinear semigroup method of Crandall and Liggett.
In this talk, we would like to present a couple of examples of stochastic MSPDEs, discuss various notions of solutions, and sketch the related methods. The central divergence-type multi-valued example is the so-called *stochastic total variation flow* on $L^2(\Lambda)$, $\Lambda \subset \mathbb{R}^d$ which is non-coercive and has a highly singular diffusivity. With additive Gaussian noise $\sqrt{Q}\,dW$ on some filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathbb{P})$ and Dirichlet boundary conditions on Λ the equation takes the following form,

$$\begin{cases} dX_t \in \operatorname{div}\left[\operatorname{sgn}(\nabla X_t)\right]\,dt + \sqrt{Q}\,dW_t \\ X_0 = x, \end{cases} \tag{2}$$

where the *sign-function* sgn becomes multi-valued for $\nabla X_t = 0$. Major applications of the involved diffusion operator can be found in image processing [10].
Existence and uniqueness of solutions and the approximation of these, as well as the approximation of their invariant measures have been demonstrated in [1,5,6] with the help of *stochastic variational inequalities*. The stochastic Cauchy-Problem on $\Lambda = \mathbb{R}^d$ is treated in [8]. A variant of (2) with linear multiplicative noise is investigated in [2]. Existence and uniqueness of solutions of a certain subclass of (1) and of (2) in all space-dimensions is proved in [7], wherein ergodicity of the Markovian semigroup of (2) is proved for space-dimensions $d = 1, 2$. In an Orlicz-space setting and with the help of a recent result of Komorowski, Peszat and Szarek [9], we also prove the ergodicity of the *stochastic "symmetrized" plasma diffusion* in $d = 1, 2$, previously treated in [3].

References

[1] V. Barbu, G. Da Prato, and M. Röckner, *Stochastic nonlinear diffusion equations with singular diffusivity*, SIAM J. Math. Anal. **41** (2009), no. 3, 1106–1120.

[2] V. Barbu and M. Röckner, *Stochastic variational inequalities and applications to the total variation flow perturbed by linear multiplicative noise*, Arch. Rational Mech. Anal., in press (2013), 38 pp., doi: 10.1007/s00205-013-0632-x.

[3] J.G. Berryman and C.J. Holland, *Asymptotic behavior of the nonlinear diffusion equation* $n_t = (n^{-1}n_x)_x$, J. Math. Phys. **23** (1982), no. 6, 983–987.

[4] I. Ciotir, *Existence and uniqueness of solution for the stochastic nonlinear diffusion equation of plasma*, Preprint (2011), 21 pp., arXiv:1103.2715.

[5] I. Ciotir and J.M. Tölle, *Convergence of invariant measures for singular stochastic diffusion equations*, Stoch. Proc. Appl. **122** (2012), no. 4, 1998–2017.

[6] I. Ciotir and J.M. Tölle, *Corrigendum to "Convergence of invariant measures for singular stochastic diffusion equations" [Stochastic Process. Appl. 122 (2012) 1998–2017]*, Stochastic Process. Appl. **123** (2013), no. 3, 1178–1181.

[7] B. Gess and J.M. Tölle, *Multi-valued, singular stochastic evolution inclusions*, to appear in J. Math. Pures Appl. (2013), 36 pp.

[8] J.U. Kim, *On a stochastic singular diffusion equation in* R^d, J. Funct. Anal. **262** (2012), no. 6, 2928–2977.

[9] T. Komorowski, S. Peszat, and T. Szarek, *On ergodicity of some Markov processes*, Ann. Probab. **38** (2010), no. 4, 1401–1443.

[10] L.I. Rudin, S. Osher, and E. Fatemi, *Nonlinear total variation based noise removal algorithms*, Physica D **60** (1992), no. 1–4, 259–268.

Martin VENKER

New Results on Local Universality of Repulsive Particle Systems

I will review recent results on the appearance of random matrix distributions in a class of repulsive particle systems on the real line. The focus will be on local correlation statistics. As a new result, I will show edge universality, meaning that the universal edge correlations from (Hermitian) random matrix theory also show up for these more general particle ensembles. Moreover, the fluctuations of the largest particle are given by the Tracy-Widom distribution. This is joint work with Friedrich Götze and Thomas Kriecherbauer.

Feng-Yu WANG

Equivalent Harnack and Gradient Inequalities for Pointwise Curvature Lower Bound

By using a coupling method, an explicit log-Harnack inequality with local geometry quantities is established for (sub-Markovian) diffusion semigroups on a Riemannian manifold (possibly with boundary). This inequality as well as the consequent L^2-gradient inequality, are proved to be equivalent to the pointwise curvature lower bound

condition together with the convexity or absence of the boundary. Some applications of the log-Harnack inequality are also introduced.

Mihail ZERVOS

Irreversible capacity expansion with possible default

We consider the problem of determining the optimal capacity expansion strategy that a firm operating within a random economic environment should adopt. We model market uncertainty by means of a general one-dimensional positive diffusion with possible absorption at 0. The objective is to maximise a performance criterion that involves a general running payoff function and associates a proportional cost with each capacity increase up to the first hitting time of 0. The resulting optimisation problem takes the form of a degenerate two-dimensional singular stochastic control problem that we explicitly solve.

Rongchan ZHU

Stochastic semilinear equations and their associated Fokker-Planck equations

The main purpose of this talk is to prove existence and uniqueness of (probabilistically weak and strong) solutions to stochastic differential equations (SDE) on Hilbert spaces under a new approximation condition on the drift, recently proposed in [1] to solve Fokker-Planck equations (FPE), extended in this paper to a considerably larger class of drifts. As a consequence we prove existence of martingale solutions to the SDE (whose time marginals then solve the corresponding FPE). Applications include stochastic semilinear partial differential equations with white noise and a non-linear drift part which is the sum of a Burgers-type part and a reaction diffusion part. The main novelty is that the latter is no longer assumed to be of at most linear, but of at most polynomial growth. This case so far had not been covered by the existing literature. We also give a direct and more analytic proof for existence of solutions to the corresponding FPE, extending the technique from [1] to our more general framework, which in turn requires to work on a suitable Gelfand triple rather than just the Hilbert state space.

References

[1] V. Bogachev, G. Da Prato, M. Röckner, Existence and uniqueness of solutions for Fokker-Planck equations on Hilbert spaces, J.Evol.Equ. 10 (2010),487-509

Xiangchan ZHU

Local existence and non-explosion of solutions for semilinear stochastic equations driven by multiplicative noise

In this paper we prove the local existence and uniqueness of solutions for a class of semilinear stochastic equations driven by multiplicative noise. We also establish that, for such a class of equations the addition of linear multiplicative noise provides a regularizing effect: the solutions will not blow up with high probability if the initial data is sufficiently small, or if the noise coefficient is sufficiently large.

Gordan ŽITKOVIĆ

Facelifting in mathematical finance

Superreplication of contingent claims in incomplete markets often involves a 'facelift': the payoff function is replaced by an envelope in an appropriate class and the new, facelifted, payoff is priced using complete-market methods. More generally, in optimal stochastic control theory, the facelift appears in the form of a discontinuity of the value function at the terminal time and typically arises when the control set is unbounded. In that case, the Hamiltonian may take infinite values and the facelift typically consist of replacing the terminal payoff by a smallest finite-Hamiltonian majorant.
When utility-based pricing is used instead of superreplication, no facelift is expected, thanks to the smoothing effect of the utility function. Indeed, the Hamiltonian applied to the terminal payoff is always finite. Yet, there is a facelift as soon as the claim being priced is non-replicable. We show this unexpected fact using control-theoretic methods and relate it to the appearance of finitely-additive dual minimizers in the problem of utility maximization with a random endowment. We also analyze the corresponding Hamilton-Jacobi-Bellman equation and provide necessary analytic conditions on its terminal condition for the absence of a facelift. This is joint work with Kasper Larsen and Mete Soner.

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00–9:45	*V. Betz*	*Y. Song*	*G. Žitković*	*S. Federico*	*F. Gong*
9:45–10:30	*Z.-M. Ma*	*S. Ji*	*P. Bank*	*N. Glatt-Holtz*	*F. Herzberg*
10:30–11:15	*S. Luo*	*X. Guo*	*L. Denis*	*B. Gess*	*F. Santambrogio*
☕ *– Coffee Break –*					
11:45–12:30	*T. Guhr*	*G. Peskir*	*B. Acciaio*	*Z. Hu*	*C. Chen*
12:30–13:15	*Z. Burda*	*P. Cheredito*	*A. Sulem*	*F. Wang*	*G. Da Prato*

– Lunch Break –

Parallel Monday: T2-137,T2-208,T2-238

Parallel Wednesday: U2-216,U2-223,U2-228

	Monday	Tuesday		Wednesday		Thursday	Friday
14:30–15:15	*B. Gentz*		*14:30–14:55*	*Parallel 4*	*14:30–15:15*	*S. Tang*	*F. Russo*
15:15–16:00	*M. Venker*	*Excursion*	*14:55–15:20*	*Parallel 5*	*15:15–16:00*	*J. Tölle*	*S. Albeverio*
☕ *– Coffee Break –*							
16:40–17:05	*Parallel 1*		*16:00–16:45*	*S. Dayanik*	*16:30–17:15*	*W. Stannat*	
17:05–17:30	*Parallel 2*		*16:45–17:30*	*M. Zervos*	*17:15–18:00*	*S. Hamadene*	
17:30–17:55	*Parallel 3*		*17:30–18:15*	*J. Obloj*			

– Conference Dinner –