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Global Estimates for Green's Functions and the Conditional Gauge

We intend to discuss global pointwise estimates for kernels of the resolvent $(I - T)^{-1}$ of integral operators $Tf(x) = \int_\Omega K(x,y)f(y)d\omega(y)$ on $L^2(\Omega,\omega)$ for $||T|| < 1$ under the only assumption that $d(x,y) = 1/K(x,y)$ is a quasi-metric. An instructive example is the dyadic kernel $K(x,y) = \sum_{Q\in\mathcal{Q}} c_Q\, \chi_Q(x)\, \chi_Q(y)$ where $\mathcal{Q}$ is the family of all dyadic cubes in $\mathbf{R}^n$, χ_Q is the characteristic function of Q, and $c_Q \geq 0$. As an application, we give sharp bilateral bounds for Green's function and Martin's kernel of the fractional Schrödinger operator $(-\triangle)^{\alpha/2} - q$ with an arbitrary nonnegative potential q (possibly a measure) on $\mathbf{R}^n$ for $0 < \alpha < n$, or a bounded NTA domain Ω for $0 < \alpha \leq 2$. This yields explicit bounds for the conditional gauge:

$$e^{\mathbf{E}_y^x\left[\int_0^\zeta q(X_t)\,dt\right]} \leq \mathbf{E}_y^x\left[e^{\int_0^\zeta q(X_t)\,dt}\right] \leq e^{C\,\mathbf{E}_y^x\left[\int_0^\zeta q(X_t)\,dt\right]},$$

for Brownian motion or α-stable Lévy processes X_t. Here ζ is the lifetime, and $\mathbf{E}_y^x$ the expectation of the conditioned process starting at x and exiting at y. The upper bound is new even in the classical case $\alpha = 2$. Other applications include necessary and sufficient conditions for the existence of weak solutions, along with sharp pointwise estimates of solutions for some nonlinear elliptic equations with natural growth terms.

This is joint work with Michael Frazier and Fedor Nazarov.