Nonlinear parabolic equations related to infinite particle systems

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1. Concept of a physically safe reactor

Let us take an ingot of uranium-238 (U_{238}). U_{238} is the largest atomic nucleus occurring naturally on earth. It has 146 neutrons compared to 92 protons.
In each cubic centimeter of ingot there are $5 \cdot 10^{22}$ atoms of U_{238}.

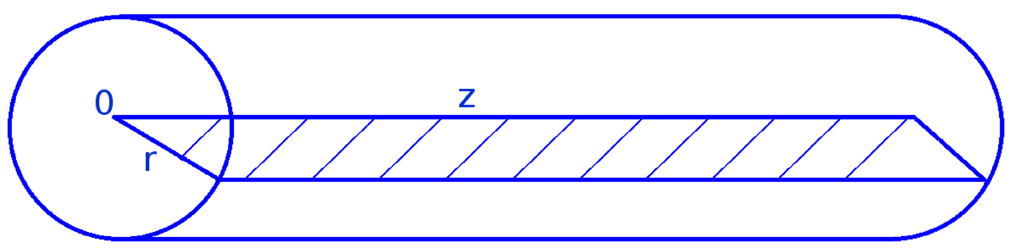

Figure 1: *Uranium ingot of radius r and length z.*

We also have an external neutron source that is located left of an uranium-238 ingot and emits neutrons. The total neutron cross section consists of scattering cross-section, absorption cross-section and fission cross-section. We are most interested in the absorption cross-section of (U_{238}). When a neutron from an external source hits an atom of U_{238} it is absorbed by the nucleus of a U_{238}. Then U_{238} becomes uranium-239 (U_{239}), an unstable element which undergoes beta decay to produce neptunium-239 (Np_{239}), which then itself decays, with a half-life of 2.355 days, into plutonium-239 (Pu_{239}).

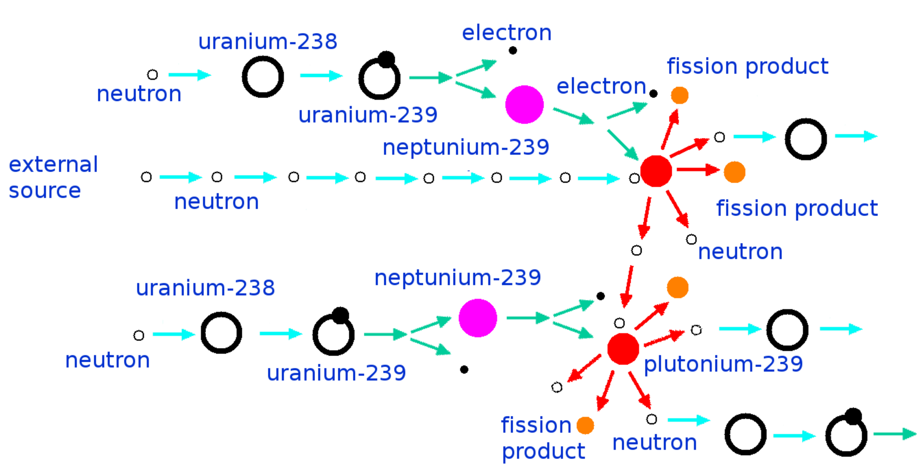

Figure 2: *Nucleon-nucleon interactions in uranium-238 ingot.*

When a neutron hits $Pu-239$ nuclear fission takes place with a high probability (fission cross-section) and two to three (on average 2,6) neutrons are released. This leads to the possibility of the newly generated neutrons inducing successive nuclear fissions

$$U_{239} \Rightarrow Np_{239} \Rightarrow Pu_{239} \Rightarrow neutrons$$

A string of such succeeding nuclear fissions induced by the generated neutrons is called a chain reaction.

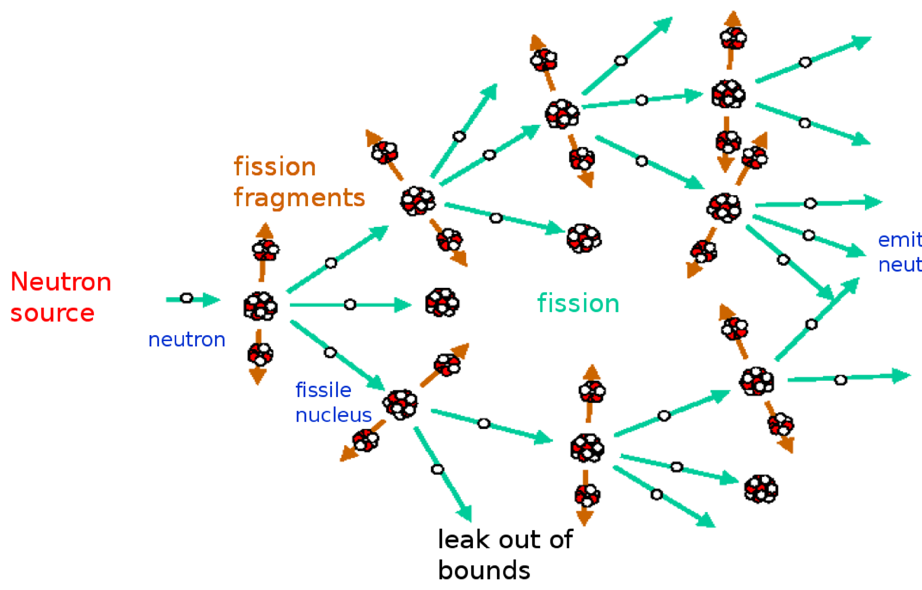

Figure 3: *Nuclear fission chain reaction.*

The burning region moves along the direction of the core axis and generates a burning wave.

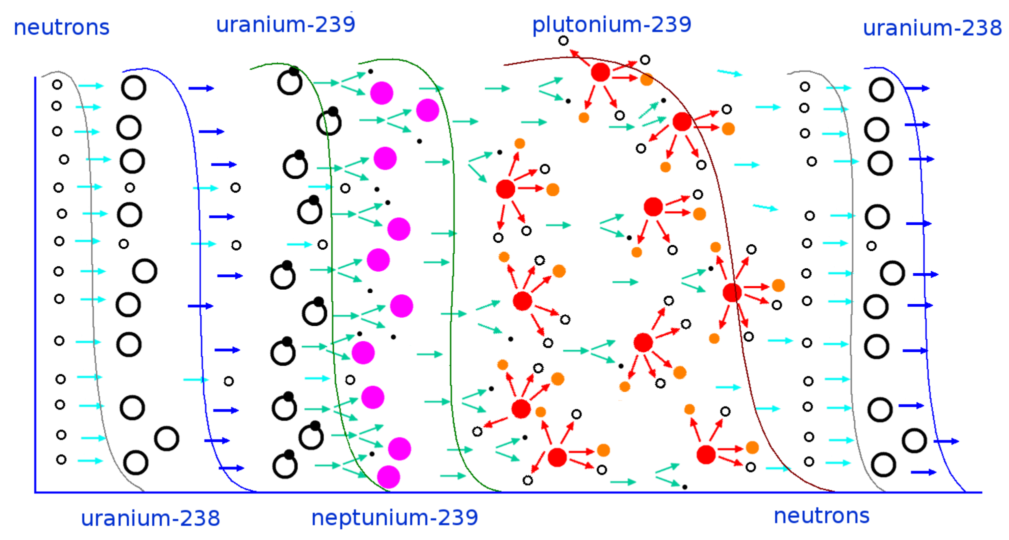

Figure 4: *Nuclear burning wave.*

2. Mathematical Explanation and Analysis Method

The neutron diffusion equations:

$$\frac{\partial n(r,z,t)}{\partial t} = D\Delta n(r,z,t) + q(r,z,t)\ ,(1),$$

$$\rho\tilde{C}\frac{\partial T(r,z,t)}{\partial t} = \aleph\Delta T(r,z,t) + q_T(r,z,t)\ .$$

Where ρ - is the density, $\tilde{C}$ - specific heat, $\aleph$ - thermal conductivity constant, D - diffusion constant of neutrons.
The nuclide transformation equations:

$$\frac{\partial N_8(r,z,t)}{\partial t} = -V_n n(r,z,t)\sigma_a^8 N_8(r,z,t)\ ,$$

$$\frac{\partial N_9(r,z,t)}{\partial t} = V_n n(r,z,t)\sigma_a^8 N_8(r,z,t) - \frac{1}{\tau_\beta}N_9(r,z,t)\ ,$$

$$\frac{\partial N_{Pu}(r,z,t)}{\partial t} = \frac{1}{\tau_\beta}N_9(r,z,t) - V_n n(r,z,t)(\sigma_a^{Pu}+\sigma_f^{Pu})N_{Pu}(r,z,t)\ ,$$

$$\frac{\partial \tilde{N}_i(r,z,t)}{\partial t} = p_i V_n n(r,z,t)\sigma_f^{Pu}N_{Pu}(r,z,t) - \frac{\ln 2\tilde{N}_i}{T_{1/2}^i}\ , i=1,6,$$

$$\frac{\partial N_{FP}(r,z,t)}{\partial t} = (1-p_i)V_n n(r,z,t)\sigma_f^{Pu}N_{Pu}(r,z,t) + \frac{\ln 2\tilde{N}_i}{T_{1/2}^i}.$$

where $n(r,z,t)$ - neutron density, V_n - neutron velocity,N_8,N_9,N_{Pu},N_{FP} - concentration of U_{238}, U_{239}, $Pu-239$ and fission products accordingly, $\tilde{N}_i$ - concentration of neutron-rich fission fragments, σ_a,σ_f - absorption cross-section and fission cross-section of neutron.

3. Simulation results obtained for one-group approximation

Kinetics of concentration U_{238}, U_{239}, $Pu-239$ and fission products on axis of the cylinder, with the initial conditions: $R = 125cm$, $Z = 1000m$, $\frac{N_{Pu}}{N_8} = \frac{1}{99}$

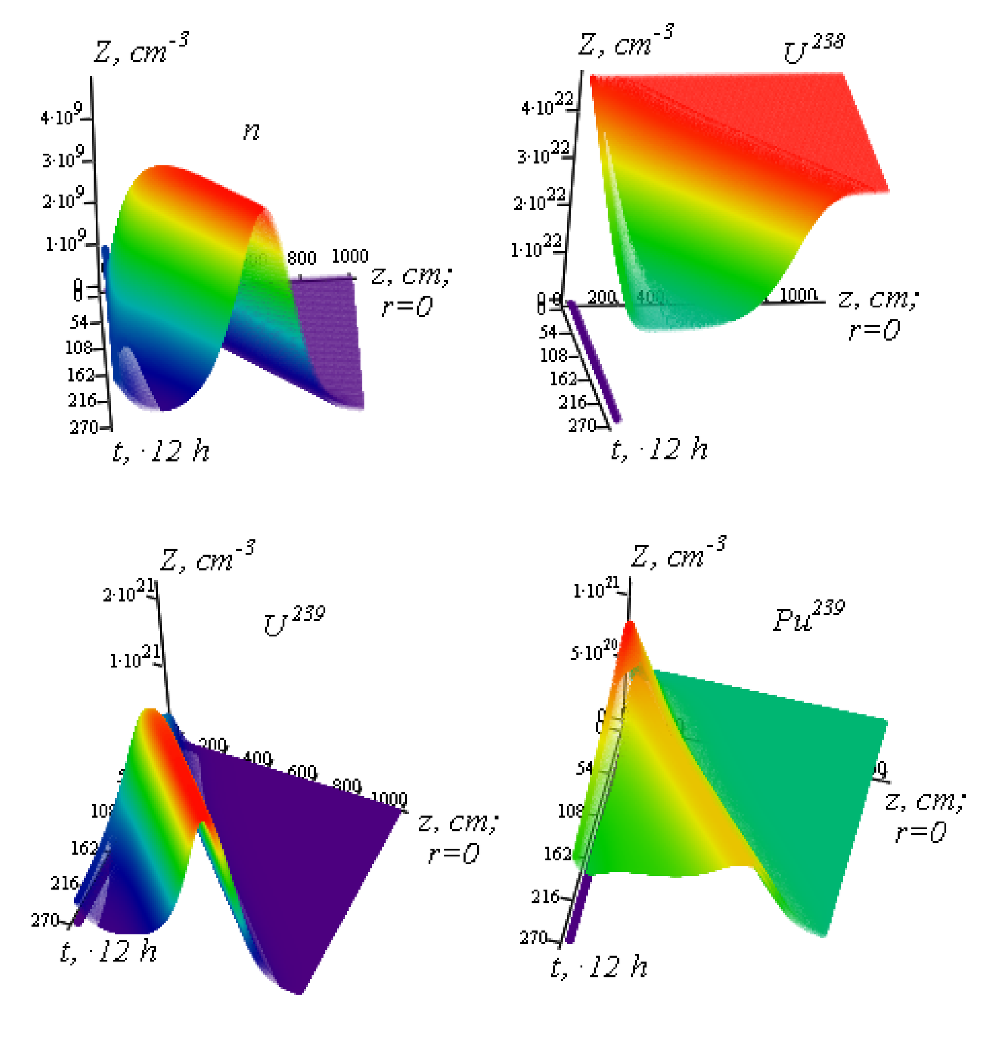

Figure 5: $t = 135$ *days*

Kinetics of concentration U_{238}, U_{239}, $Pu-239$ and fission products in the cylinder, with the initial conditions: $R = 100cm$, $Z = 800m$, $\frac{N_{Pu}}{N_8} = \frac{2}{98}$

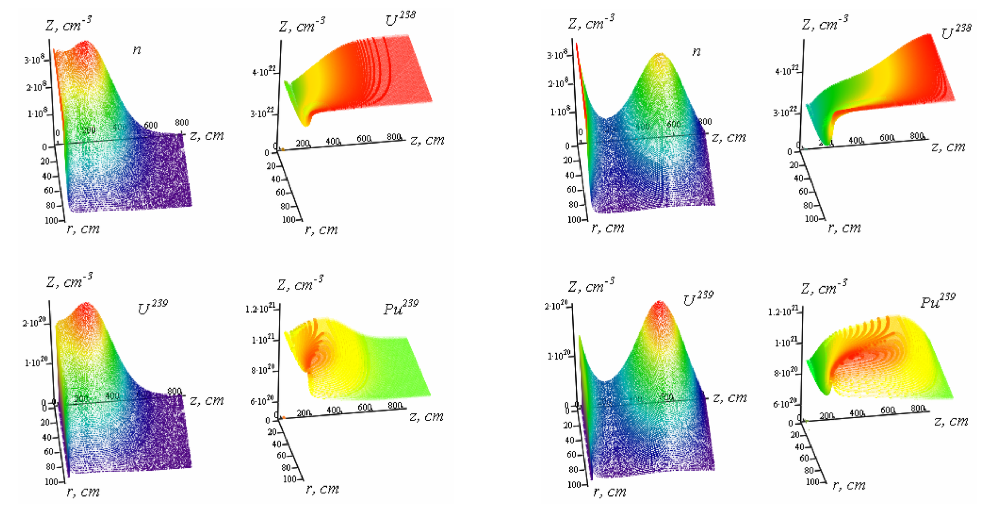

Figure 6: $t_1 = 110$ *days and* $t_2 = 210$ *days*

It's easy to see nuclear burning wave that appeared on the graphics of concentration of neutrons, U_{238}, U_{239} and $Pu-239$.

4. Analytical solution

An analytical solution is obtained for three-dimensional one-group diffusion model. We can rewrite (1) as:

$$\frac{1}{v}\frac{\partial\phi}{\partial t} = \nabla\cdot(D\nabla\phi) + (k_\infty - 1 - \gamma\phi)\Sigma_\phi;$$

where ϕ - is the neutron flux, v - the neutron velocity, D - the diffusion coefficient, Σ_a - the macroscopic absorption cross-section, k_∞ - characterizes the multiplication factor in an infinite medium at zero power condition and is fluency dependent, and the term γ - with negative γ is introduced for modeling reactivity feedback effects in an approximate manner. The boundary condition is, neutron flux D and Σ_a are assumed to be constant. The burn-up effect is described reasonably well by:

$$k_\infty = k_\infty(\psi)$$

where ψ denotes the neutron flux, defined as

$$\psi = \int_{-\infty}^{t}\phi(x,y,z,t')\partial t'$$

for an asymptotic problem.
The burn-up is assumed to be a parabolic function of ψ

$$k_\infty = k_{max} - (k_{max} - k_0)(\frac{\psi}{\psi_m} - 1)^2$$

The traveling wave solution was looked for in the form of

$$\phi(z,t) = \phi(\xi)\ ,\xi = z - ut,$$

where u is the burn-up wave drift speed and is much smaller than v.
The singleton wave solution was obtained in the form of

$$\phi(\xi) = M^2 N sech^2(MN\xi - D)$$

5. Conclusion

The realization of a self controlled reactor was proposed. Numerical solution for the neutron diffusion equations and nuclide transformation equations was obtained. Analytical solution is obtained. The program for the burnup process simulation taking into account boundary and initial conditions was written. Reactor activation and formation of nuclear burning wave (in particular, Feoktistov neutron-fission progressive wave) were demonstrated. The multi-group solution for four groups of neutrons is in progress.